

TeV Scale $B - L$ model with a flat Higgs potential at the Planck scale - in view of the hierarchy problem -

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Abstract

The recent discovery of the Higgs-like particle at around 126 GeV has given us a big hint towards the origin of the Higgs potential. Especially the running quartic coupling vanishes near the Planck scale, which indicates a possible link between the physics in the electroweak and the Planck scales. Motivated by this and the hierarchy problem, we investigate a possibility that the Higgs has a flat potential at the Planck scale. In particular, we study the RG analysis of the $B - L$ extension of the standard model [1] with a classical conformality. The $B - L$ symmetry is radiatively broken at the TeV scale via the Coleman-Weinberg mechanism. The electroweak symmetry breaking is triggered by a radiatively generated scalar mixing so that its scale 246 GeV is dynamically related with the $B - L$ breaking scale at TeV. The Higgs boson mass is given at the border of the stability bound, which is lowered by a few GeV from the SM by the effect of the $B - L$ gauge interaction.

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1 Introduction

The dynamics of the electroweak symmetry breaking (EWSB) and the origin of the Higgs potential are the most important issues in the standard model (SM). The ATLAS and CMS groups have announced a discovery of the Higgs-like particle at around 126 GeV [2, 3]. This value of 126 GeV is quite suggestive to the physics at very high energy since it is close to the border of the vacuum stability bound up to the Planck scale. In the SM, the Higgs mass is determined by the quartic coupling λ_H of the Higgs field. For a relatively light Higgs boson, the β function of λ_H becomes negative and the running coupling λ_H crosses zero at some high energy scale. This implies an instability of the Higgs potential. The theoretical investigation [4] shows that the stability bound up to the Planck scale requires (see also [5] for larger uncertainties of the top mass)

$$M_h[\text{GeV}] > 129.4 + 1.4 \left(\frac{M_t[\text{GeV}] - 173.1}{0.7} \right) - 0.5 \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 1.0_{th}. \quad (1)$$

The observed mass of 126 GeV is close to but lower by a few GeV from the above value of the stability bound in the SM. If the Higgs mass is lighter than the above bound, new physics must appear below the Planck scale. But if it is just at the border of the stability bound, it may give a big hint to the origin of the Higgs potential at the Planck scale [6, 7, 8, 9].

Another important clue to the Higgs potential comes from the hierarchy problem, i.e. the stability of the Higgs mass against higher energy scales such as the GUT or the Planck scale. The most natural solution is the low energy supersymmetry, but the new physics search at the LHC has put stringent constraints on simple model constructions and a large parameter region of the TeV scale supersymmetric models has been already excluded. Of course, we cannot rule out a possibility to find an indication of the low energy supersymmetry in the near future, but it will be important to reconsider the hierarchy problem from a different point of view.

In this paper, we take an alternative approach to the hierarchy problem following the Bardeen's argument [10]. In section 2, we give an interpretation of his argument in terms of the renormalization group equations (RGE). If we adopt it, the most natural mechanism to break the electroweak symmetry is the Coleman-Weinberg (CW) mechanism [11]. In section 3, we emphasize that the CW mechanism is another realization of a dimensional transmutation, and stable against higher energy scales. It is, however, well known that the CW mechanism does not work within the SM because of the large top Yukawa coupling. Hence we need to extend the SM. In section 4, we introduce our model, a classically conformal $B-L$ extension of the SM [1]. The anomaly free global symmetry of the SM, $B-L$ (baryon number minus lepton number), is gauged, and the right-handed neutrinos and a SM singlet scalar Φ are introduced. This model has a classical conformal invariance [12], namely there are no explicit mass terms

in the scalar potential. We furthermore assume, motivated by the 126 GeV Higgs, that the Higgs potential is flat at the Planck scale. In section 5, we discuss the dynamics of the model using the RGEs. We first study the radiative breaking of the $B - L$ gauge symmetry via the CW mechanism. Then we show that a small negative value of the mixing $\lambda_{mix}(H^\dagger H)(\Phi^\dagger \Phi)$ is radiatively generated by solving the RGEs. This triggers the EWSB. We then discuss the predictability of the model. We also show that the stability bound of the Higgs potential up to the Planck scale is lowered than the SM prediction by a few GeV by the $B - L$ gauge interaction. Finally we conclude in section 6.

2 Bardeen's argument on the hierarchy problem

We pay a special attention to the almost scale invariance of the SM. At the classical level, the SM Lagrangian is conformal invariant except for the Higgs mass term. Bardeen has argued [10] that once the classical conformal invariance and its minimal violation by quantum anomalies are imposed on the SM, it may be free from the quadratic divergences. Bardeen's argument on the hierarchy problem is interpreted as follows [13]. In field theories, we have two kinds of divergences, logarithmic and quadratic divergences. The logarithmic divergence is operative both in the UV and the IR. In particular, it controls a running of coupling constants and is observable. On the other hand, the quadratic divergence can be always removed by a subtraction. Once subtracted, it no longer appears in observable quantities. In this sense, it gives a boundary condition of a quantity in the IR theory at the UV energy scale where the IR theory is connected with a UV completion theory. Indeed, the RGE of a Higgs mass term m^2 in the SM

$$V(H) = -m^2 H^\dagger H + \lambda_H (H^\dagger H)^2 \quad (2)$$

is approximately given by

$$\frac{dm^2}{dt} = \frac{m^2}{16\pi^2} \left(12\lambda_H + 6Y_t^2 - \frac{9}{2}g^2 - \frac{3}{2}g_Y^2 \right). \quad (3)$$

Y_t is the top Yukawa coupling and g, g_Y are $SU(2)_L, U(1)_Y$ gauge couplings. The quadratic divergence is adjusted by a boundary condition either at the IR or UV scale. Once the initial condition of the RGE is given at the UV scale, it is no longer operative in the IR. The RGE shows that the mass term m^2 is multiplicatively renormalized. If it is zero at a UV scale M_{UV} , it continues to be zero at lower energy scales. In this sense, the quadratic divergence is not the issue in the IR effective theory, but the issue of the UV completion theory.

The multiplicative renormalizability of the mass term is violated by a presence of a mixing with another scalar field Φ

$$V_{mix}(H, \Phi) = \lambda_{mix}(H^\dagger H)(\Phi^\dagger \Phi). \quad (4)$$

Then the RGE is modified as

$$\frac{dm^2}{dt} = \frac{m^2}{16\pi^2} \left(12\lambda_H + 6Y_t^2 - \frac{9}{2}g^2 - \frac{3}{2}g_Y^2 \right) + \frac{M^2}{8\pi^2} \lambda_{mix} \quad (5)$$

where M is the mass of the Φ field. The last term comes from the logarithmic divergence

$$\delta m^2 \sim \frac{\lambda_{mix} M^2}{16\pi^2} \log(M^2/m^2) \quad (6)$$

due to the loop diagram of the scalar particle Φ . Therefore, the hierarchy problem, namely the stability of the EWSB scale, is caused by such a mixing of relevant operators (mass terms) with hierarchical energy scales $m \ll M$.

From the above considerations, we can divide the hierarchy problem into the following two different issues;

- Boundary condition of dimensionful parameters (such as m^2) at M_{UV}
- Mixing of relevant operators as in (5)

The first is related to the quadratic divergences while the second to the logarithmic divergences.

Supersymmetry is most favored in solving the hierarchy problem. If its breaking scale is not so high, it can solve both issues of the hierarchy problem. It is beautiful, but the recent experiments have put severe constraints on the model constructions with the low energy supersymmetry. But we do not need to solve both issues simultaneously. Quadratic divergences are subtracted at the UV cut-off scale as a boundary condition. The justification is necessary in the UV completion theory. On the contrary, in order to avoid the operator mixings with high energy scales, we need to impose an absence of intermediate scales between TeV and Planck scales¹. This is emphasized in the Bardeen's argument [10]. Then the Planck scale physics is directly connected with the electroweak physics. Such a view has been emphasized also by Shaposhnikov [14, 15]. A natural boundary condition of the mass term at the UV cut-off scale, e.g. M_{Pl} , is

$$m^2(M_{Pl}) = 0. \quad (7)$$

¹This requires that the grand unification, if exists, occurs at the Planck scale. It is interesting to construct phenomenologically viable models in string theory that both of the supersymmetries and the grand unification are broken at the Planck scale.

This is the condition of the classical conformality. The condition (7) must be justified in the UV completion theory. From the low energy effective theory point of view, it is just imposed as a boundary condition.

3 Stability of the Coleman-Weinberg mechanism

If there are no intermediate scales, mass parameters are multiplicatively renormalized. Then, if we set the dimensionful mass parameters zero at the UV scale, they continue to be absent in the low energy scale. Such a model is called a *classically conformal* model [12]. Conformal invariance is broken by a logarithmic running of the coupling constants, but no explicit mass terms arise by radiative corrections. Hence, the EWSB must be realized not by the negative mass squared term of the Higgs doublet but by the radiative breaking such as the Coleman-Weinberg (CW) mechanism [11]. In this section we see the stability of the symmetry breaking scale in the CW mechanism against higher energy scales.

In the SM, we have two typical mass scales, QCD and the electroweak scales. Let us compare the emergence of a low energy scales in the CW mechanism and QCD. QCD scale Λ_{QCD} is dynamically generated at a low energy where the running coupling constant diverges. It is given as a function of the coupling $\alpha_s(M_{UV})$ at a UV scale M_{UV} by

$$\Lambda_{QCD} = M_{UV} \exp \left(-\frac{2\pi}{b_0 \alpha_s(M_{UV})} \right). \quad (8)$$

b_0 is the coefficient of the β function $\beta = d\alpha_s/dt = -(b_0/2\pi)\alpha_s^2$. Since the β function is proportional to \hbar , the small QCD scale $\sim M_{UV} \exp(-c/\hbar)$ is nonperturbatively generated, and stable against radiative corrections of higher energy scales.

Similarly, if the EWSB is realized by the CW mechanism, its breaking scale emerges radiatively from the coupling constant at a UV scale. In comparison to the dimensional transmutation in QCD, the symmetry breaking scale M_{CW} emerges near the scale where the running coupling constant crosses zero. In order to realize the zero-crossing of the running coupling constant, the β function must take a positive value $\beta > 0$ near the breaking scale M_{CW} . Let's make an approximation that $\beta = b > 0$ is constant for simplicity. See, e.g. eq.(21). Then the running coupling constant is approximately given by

$$\lambda(t) = b(t - t_0) = b(t - t_{UV}) + \lambda_{UV}, \quad (9)$$

where we have introduced the boundary condition $\lambda(t_{UV}) = \lambda_{UV}$. The running coupling $\lambda(t)$ vanishes at $t = t_0 = t_{UV} - \lambda_{UV}/b$. The renormalized effective potential of the scalar field ϕ

with a quartic self-coupling λ is given by

$$V(\phi) = \frac{1}{4}\lambda(t)\phi^4 = \frac{bM^4}{4}(t - t_0)e^{4t} \quad (10)$$

where $t = \log[\phi/M]$ and M is the renormalization point. (We have neglected the anomalous dimension of the field for simplicity.) The potential has a minimum at $t = t_0 - 1/4$. Hence, the breaking scale $M_{CW} = \langle\phi\rangle$ is given by

$$M_{CW} = M_{UV} \exp\left(-\frac{\lambda_{UV}}{b} - \frac{1}{4}\right). \quad (11)$$

The emergence of the scale M_{CW} is similar to the dimensional transmutation (8) in QCD. The exponent of the rhs in eq.(11) shows a balance between the contribution to the effective potential from the tree-level coupling λ_{UV} and the loop contribution proportional to b (and \hbar). Such a balance is necessary for the CW mechanism to occur. In particular, as emphasized in [11], the CW mechanism does not occur in a scalar QED without the gauge interaction. A small value of b can generate a small energy scale M_{CW} from a very high energy scale M_{UV} . In this sense, the CW mechanism is similar to the dimensional transmutation in QCD and stable against the higher energy scales. It is the reason why the CW mechanism can be an alternative solution to the gauge hierarchy problem.

4 Classically conformal $B - L$ model

In the SM, the dominant contribution to the β function of the Higgs quartic coupling comes from the gauge couplings, top Yukawa coupling and the quartic coupling itself,

$$\beta_H = \frac{1}{16\pi^2} \left(24\lambda_H^2 - 6Y_t^4 + \frac{9}{8}g^4 + \frac{3}{8}g_Y^4 \right). \quad (12)$$

In order to realize the CW mechanism, the β function must take a positive value. It is, however, well-recognized that the large top-Yukawa coupling Y_t makes it negative and the CW mechanism does not work in the SM. Hence, in order to break the EW symmetry radiatively, we need to extend the SM so that the CW mechanism works with phenomenologically viable parameters.

The idea to utilize the CW mechanism to solve the hierarchy problem was first modelled by Meissner and Nicolai [12]. They proposed an extension of the SM with the classically conformal invariance (see also [16, 17, 18, 19, 20, 21, 22, 23]). In addition to the SM particles, right-handed neutrinos and a SM singlet scalar Φ are introduced.

In previous papers [1], inspired by the above work [12], we have proposed a minimal phenomenologically viable model that the electroweak symmetry can be radiatively broken. It is the minimal $B - L$ model [24, 25, 26]. The model is similar to the one proposed by [12], but

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$
q_L^i	3	2	$+1/6$	$+1/3$
u_R^i	3	1	$+2/3$	$+1/3$
d_R^i	3	1	$-1/3$	$+1/3$
ℓ_L^i	1	2	$-1/2$	-1
ν_R^i	1	1	0	-1
e_R^i	1	1	-1	-1
H	1	2	$-1/2$	0
Φ	1	1	0	$+2$

Table 1: Particle contents of minimal $B - L$ model (except for the gauge bosons). In addition to the SM particles, the right-handed neutrino ν_R^i ($i = 1, 2, 3$ denotes the generation index) and a complex scalar Φ are introduced.

the difference is whether the $B - L$ symmetry is gauged or not. We showed that the gauging of $B - L$ symmetry plays an important role to achieve the radiative $B - L$ symmetry breaking. It is also phenomenologically favorable.

4.1 The model

A classically conformal $B - L$ extension of the SM is based on the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. The particle contents are listed (except for the gauge bosons) in Table 1. In addition to the SM particles, the model consists of the $B - L$ $U(1)$ gauge field, a SM singlet scalar Φ and right handed neutrinos ν_R^i . The three generations of right-handed neutrinos (ν_R^i) are necessary to make the model free from all the gauge and gravitational anomalies. The Lagrangian relevant for the seesaw mechanism is given by

$$\mathcal{L} \supset -Y_D^{ij} \overline{\nu_R^i} H^\dagger \ell_L^j - \frac{1}{2} Y_N^i \Phi \overline{\nu_R^i} \nu_R^i + \text{h.c.}, \quad (13)$$

where the first term gives the Dirac neutrino mass term after the EWSB, while the right-handed neutrino Majorana mass term is generated through the second term associated with the $B - L$ symmetry breaking. Without loss of generality, we here work on the basis where the second term is diagonalized and Y_N^i is real and positive.

The gauge couplings are introduced in the covariant derivative²

$$D_\mu \phi = \partial_\mu \phi + i[g_Y Q^Y B_\mu + (g_{mix} Q^Y + g_{B-L} Q^{B-L}) B_\mu^{B-L}]. \quad (14)$$

Here B_μ and B_μ^{B-L} are gauge bosons of $U(1)_Y$ and $U(1)_{B-L}$, and Q^Y , Q^{B-L} are their charge operators. These $U(1)$ gauge bosons get mixed through loop corrections. The scale at which

²Since there are multiple matter fields coupled differently with the two gauge fields in the present model, we need to introduce g_{mix} as an independent parameter. See [27] for a general discussion.

the $U(1)$ gauge mixing becomes zero is introduced as a new parameter. The magnitude of the running coupling g_{mix} is mostly determined by other gauge couplings g_Y and g_{B-L} . In the previous analysis of the model in [1], we neglected the effect of the gauge mixing between $U(1)_{B-L}$ and $U(1)_Y$. Since we are interested in the radiative mixing of the scalars, Φ and H , the mixing of $U(1)$ gauge bosons plays a very important role in this paper.

4.2 Higgs potential

Under the hypothesis of the classically conformal invariance, the scalar potential is given by

$$V(\Phi, H) = \lambda_H(H^\dagger H)^2 + \lambda_\Phi(\Phi^\dagger \Phi)^2 + \lambda_{mix}(\Phi^\dagger \Phi)(H^\dagger H). \quad (15)$$

In [1], we chose these 3 quartic couplings by hand so that the $B-L$ and EW symmetries are spontaneously broken at TeV and EW scales. Especially it was necessary to take the mixing λ_{mix} to be a small negative value $\sim (-10^{-3})$, which seems quite artificial. In this paper, we show that such a small negative scalar mixing can be radiatively generated if we assume that the Higgs has a flat potential at a UV scale (e.g. Planck scale).

Now we explain the most important assumption in the paper. Motivated by the light Higgs boson mass around 126 GeV, we impose a simple assumption that

$$\textit{The Higgs has a flat potential at a UV scale, } M_{UV}.$$

The vanishing quartic coupling λ_H at a high energy scale is suggested by the experimental indication of the light Higgs boson mass around 126 GeV. Namely, the running coupling $\lambda_H(t)$ crosses zero at a UV scale, and the vacuum becomes unstable. In [4], a detailed investigation is presented, and the Higgs potential is shown to develop an instability around 10^{11} GeV for the Higgs mass 124 – 126 GeV. However, because of the very slow running of the Higgs quartic coupling at a higher energy scale, the instability scale is very sensitive to theoretical and experimental uncertainties, and the stability up to the Planck scale cannot be excluded (see recent papers [5]). Here we take the light Higgs boson mass as an indication of a vanishing quartic coupling at the UV scale M_{UV} .

In addition to it, we further assume that the scalar mixing $\lambda_{mix}H^\dagger H|\Phi|^2$ vanishes at M_{UV} . Then the potential $V(\Phi, H)$ is completely flat into the direction of H and becomes

$$V(\Phi, H)|_{UV} = \lambda_\Phi(\Phi^\dagger \Phi)^2 \quad (16)$$

at the UV scale M_{UV} . We will show in the following section that radiative corrections generate a small negative value of the scalar mixing $\lambda_{mix} \sim -10^{-3}$ at a lower energy scale. If the $B-L$ symmetry is broken, its VEV $\langle \Phi \rangle$ triggers the EWSB. Hence the scales of $B-L$ breaking and the EWSB are related. The square root of the mixing $|\lambda_{mix}|$ gives a ratio between these two scales.

Because of the classically conformal invariance and the assumption that the Higgs has a flat potential at the UV scale, the model is characterized by a very few parameters. Besides the SM couplings and the Yukawa couplings of ν_R , the model has three additional parameters

1. $B - L$ gauge coupling (g_{B-L})
2. SM singlet quartic coupling (λ_Φ)
3. Energy scale at which g_{mix} vanishes.

As stated above, the magnitude of the gauge mixing is almost determined by the magnitudes of other gauge couplings and the scale at which g_{mix} vanishes is not very important in determining the dynamics. In this sense, there are only two parameters that are important in the dynamics of the model. One of them determines the scale of the EWSB. Hence the model is essentially described by only one parameter and has a high predictability (or excludability).

5 RGE analysis of the model

In this section, we look at the behaviors of the RGEs, and discuss how the symmetry breakings occur. The RGEs are given in the appendix and they can be easily solved numerically.

The classical conformality forbids the explicit breaking of the conformal invariance by dimensionful parameters. So no scalar mass terms are allowed at the UV cut-off scale. If it is absent at the UV boundary, it no longer appears at a lower energy scale as shown in eq.(5). We further impose the flat potential hypothesis. Then the scalar potential is given by eq.(15) with a boundary condition

$$\lambda_H(M_{UV}) = \lambda_{mix}(M_{UV}) = 0. \quad (17)$$

$V(\Phi, H)$ is invariant under the shift of H at the cut off scale M_{UV} . But the shift symmetry is violated by the gauge coupling of the Higgs field or Yukawa couplings. They generate the potential of the Higgs field at a lower energy scale through radiative corrections. The only parameter in the scalar potential at the Planck scale is the quartic coupling λ_Φ of the SM singlet scalar field, which determines the $B - L$ breaking scale M_{B-L} via the CW mechanism.

5.1 $B - L$ symmetry breaking

Let us first look at the behavior of the quartic coupling of the Φ field. The RGE is given by eq.(35). For appropriate parameters, the self-coupling λ_Φ is positive at a higher energy

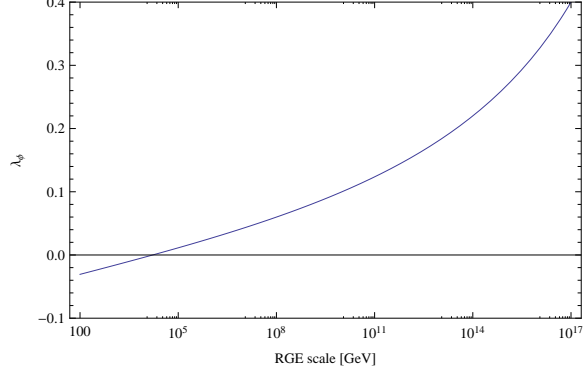


Figure 1: RG evolution of the self-coupling λ_ϕ of a SM singlet scalar ϕ . Since the β function is positive, the running coupling crosses zero at a lower energy scale.

scale and crosses zero at a lower scale. The running coupling $\lambda_\Phi(t)$ near the crossing point is proportional to the (positive-valued) β function,

$$\alpha_{\lambda,\Phi} \equiv \frac{\lambda_\Phi}{4\pi} \propto \alpha_{B-L}^2 > \frac{1}{96} \text{Tr}[\alpha_N^2]. \quad (18)$$

The inequality is required by the positiveness of the β function. If it is satisfied, Φ field gets a nonzero vev and the $B - L$ symmetry is spontaneously broken. See [1] for more details. The breaking scale is correlated with the coupling $\lambda_\Phi(M_{UV})$ at a UV cut-off. The figure 1 shows a typical behavior of the running coupling λ_Φ . For a specific choice of λ_Φ at the Planck scale, it crosses zero at a lower energy scale around $M_0 \sim 10 \text{ TeV}$. Then the $B - L$ symmetry is broken at $M_{B-L} \sim M_0 \exp(-1/4)$. As shown in [1], the SM singlet scalar has a mass

$$m_\phi^2 = \frac{6\pi}{11} \lambda_{\phi,eff} M_{B-L}^2 \quad (19)$$

where $\lambda_{\phi,eff}$ is the physical quartic coupling at the breaking scale M_{B-L} . The ratio of the scalar boson mass to the $B - L$ gauge boson mass is given [1] by

$$\left(\frac{m_\Phi}{m_{Z'}} \right)^2 \sim \frac{6}{\pi} \alpha_{B-L}. \quad (20)$$

The condition that the $B - L$ gauge coupling does not diverge up to the Planck scale requires $\alpha_{B-L} < 0.015$ at M_{B-L} . Hence the scalar boson becomes lighter than the $B - L$ gauge boson, $m_\Phi^2 < 0.03 m_{Z'}^2$. Such a very light scalar boson is a general prediction of the CW mechanism.

5.2 Electroweak symmetry breaking

The EWSB is triggered by the $B - L$ breaking. In the previous paper [1], we assumed a small negative value of the mixing λ_{mix} of H and Φ . In this paper, we show that it can be generated

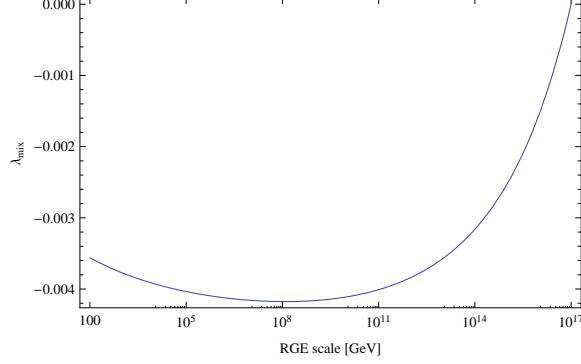


Figure 2: RG evolution of scalar mixing between a SM singlet Φ and the Higgs H . Starting from zero mixing at $M_{UV} = 10^{17} \text{ GeV}$, a small negative mixing is radiatively generated at a lower energy scale. The mixing triggers the EWSB.

radiatively by solving the RGE (36). In solving the RGE, we put a boundary condition that the mixing λ_{mix} vanishes at M_{UV} . It comes from the flatness condition of the scalar potential in the H direction at the UV cut-off scale.

RG evolutions of various couplings can be obtained numerically and the behavior of the running of the scalar mixing is drawn in figure 2. From it, we can read that a very small negative mixing is radiatively induced at IR scale. In order to understand the universality of such a behavior, we give the following approximate argument. Since $|\lambda_{mix}| \ll 1$, the RGE is approximated as

$$\frac{d\lambda_{mix}}{dt} \sim \frac{3}{4\pi^2} g_{mix}^2 g_{B-L}^2. \quad (21)$$

If there were no gauge mixing between $U(1)_Y$ and $U(1)_{B-L}$ gauge fields, the scalar mixing term would never be generated radiatively.

Let us look at the RGE of the gauge mixing (31). Since the gauge mixing term is much smaller than other gauge couplings, eq.(31) is approximated as

$$\frac{dg_{mix}}{dt} \sim \frac{2}{3\pi^2} g_{B-L} g_Y^2. \quad (22)$$

The β -function is proportional to the cube of the $B-L$ and $U(1)_Y$ gauge couplings. Hence even if the gauge mixing is absent at some scale, it is radiatively generated.

Now from eq.(21), the scalar mixing λ_{mix} is also radiatively generated through the gauge mixing. The running of the scalar mixing coupling is drawn in fig. 2. Because of the very small gauge mixing g_{mix} , the scalar mixing is much highly suppressed. The magnitude of the scalar mixing at a lower energy scale is roughly estimated from (21) and (22) as

$$\lambda_{mix} \propto -g_{mix}^2 g_{B-L}^2 \propto -g_{B-L}^4 g_Y^4. \quad (23)$$

The sign of the scalar mixing is negative at a lower energy scale because of the positive β function in (21).³

If the Φ field acquires a VEV $\langle\Phi\rangle = M_{B-L}$, the mixing term $\lambda_{mix}(H^\dagger H)(\Phi^\dagger\Phi)$ gives an effective mass term of the H field. Since the coefficient λ_{mix} is negative, the EWSB is triggered and the Higgs VEV is given by

$$v = \langle H \rangle = \sqrt{\frac{-\lambda_{mix}}{\lambda_H}} M_{B-L} \sim c \frac{\alpha_{B-L}\alpha_Y}{\sqrt{\lambda_H}} M_{B-L}. \quad (24)$$

The coefficient is dependent on the details of the running but roughly given by $c \sim 250$. This gives the ratio between the EWSB scale to the $B-L$ symmetry breaking scale.

5.3 Model predictions

The model has three additional parameters (g_{B-L} , λ_Φ and g_{mix}) besides the Yukawa couplings of the right-handed neutrinos. The Yukawa couplings do not affect the dynamics of the scalar potential very much if they are within the perturbative regime. Also the gauge mixing g_{mix} is experimentally constrained to be small at a low energy scale and its magnitude is almost determined by other gauge couplings. The other two parameters determine the $B-L$ breaking scale M_{B-L} and EWSB scale M_{EW} . The quartic coupling λ_Φ is directly related to M_{B-L} through the RGE of λ_Φ . The other coupling g_{B-L} determines the Higgs VEV through eq.(24). So two parameters λ_Φ, g_{B-L} determine the two breaking scales M_{B-L} and $M_{EW} = v$.

The experimental input $v = 246$ GeV gives a relation between the two parameters and the dynamics of the model is essentially described by a single parameter. The figure 3 shows the prediction of our model. The vertical axis is the strength of α_{B-L} and the horizontal axis is the mass of the $B-L$ gauge boson. The black line (from top left to down right) shows the prediction of our model. The model prediction can be heuristically understood as follows. If we fix the $U(1)_Y$ coupling g_Y and the Higgs quartic coupling λ_H , the input $v = 246$ GeV and the relation (24) relates the $B-L$ breaking scale M_{B-L} and the gauge coupling g_{B-L} as $M_{B-L} \sim v\sqrt{\lambda_H}/(c\alpha_{B-L}\alpha_Y)$. On the other hand, the gauge boson mass is given by $m_{Z'}^2 = 16\pi\alpha_{B-L}M_{B-L}^2$. Hence, we have a relation between the $B-L$ gauge boson mass and the gauge coupling

$$\alpha_{B-L}m_{Z'}^2 \sim (16\pi\lambda_H v^2/c^2\alpha_Y^2). \quad (25)$$

The quantities in the rhs are all known. This gives a very crude approximation of the model prediction of the TeV scale $B-L$ model with a flat potential at the UV scale M_{UV} . A more

³Note that the above argument is valid only when the RGE (36) can be approximated by (21). The figure 2 shows that the beta function changes its sign to negative at a lower energy scale. In this region, the top Yukawa coupling Y_t becomes larger there and the term proportional to $\lambda_{mix}Y_t^2(<0)$ becomes dominant.

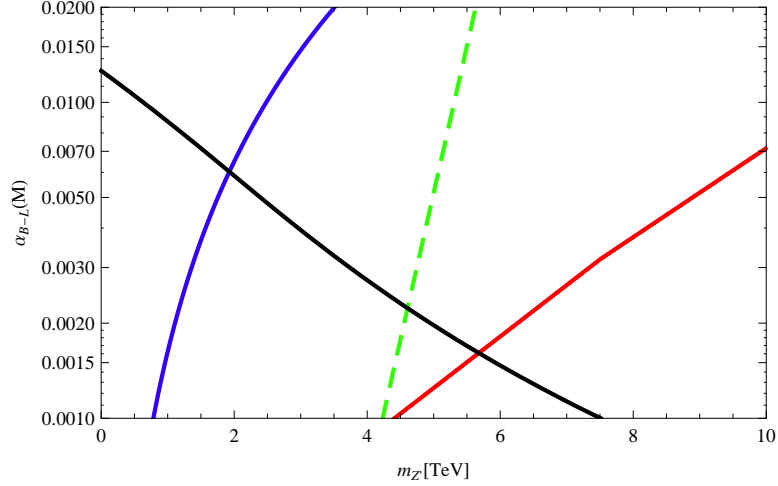


Figure 3: Model prediction is drawn in the black line (from top left to down right). The $B - L$ gauge coupling α_{B-L} and the gauge boson mass $m_{Z'}$ are related because of the *flat potential assumption at the Planck scale*. The left side of the most left solid line in blue has been already excluded by the LEP experiment. The left of the dashed line can be explored in the 5- σ significance at the LHC with $\sqrt{s}=14$ TeV and an integrated luminosity 100 fb^{-1} . The left of the most right solid line (in red) can be explored at the ILC with $\sqrt{s}=1$ TeV, assuming 1% accuracy.

precise relation is obtained, as shown in the figure 3, by solving the RGEs. We have also drawn the excluded region by LEP2 experiment, the LHC reach at 14 TeV with 100 fb^{-1} , and the ILC reach at 1 TeV.

5.4 Higgs quartic coupling and the stability bound

Finally we consider the RGE of the Higgs quartic coupling. It is assumed to be zero at the UV cut-off scale. The RGE of the Higgs quartic coupling is given by eq.(34). Compared to the SM, the β function has a contribution from the scalar mixing term λ_{mix} and the $U(1)$ gauge mixing g_{mix} . Both mixings contribute to the β function positively. Since the β function is negative in the SM for the 126 GeV Higgs mass, the effect of the $B - L$ gauging reduces the magnitude of the slope of the running coupling. Hence, the $B - L$ extension of the SM makes the vacuum stabler up to a higher energy scale than the SM. Namely, the stability bound (1) of the Higgs boson mass is lowered.

The stability bound is estimated within the 2-loop SM RGEs and the 1-loop $B - L$ RGEs including the threshold effects. The figure 4 shows the Higgs boson mass at the stability bound up to the UV cut off scale $M_{UV} = 10^{17}$ GeV as a function of the $B - L$ gauge coupling. For a stronger $B - L$ gauge coupling, the effect to the β function become larger and the Higgs boson

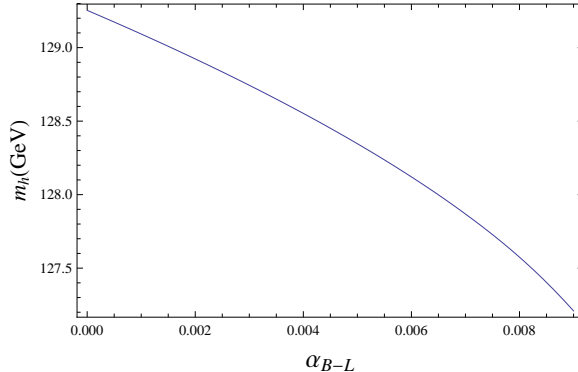


Figure 4: Higgs stability bound up to $M_{UV} = 10^{17}$ GeV as a function of the $B - L$ gauge coupling. We set $M_t = 173.1$ GeV and $\alpha_s(M_Z) = 0.1184$.

mass at the stability bound can be lowered. In the present model with a flat Higgs potential at M_{UV} , the gauge coupling and the $B - L$ gauge boson mass are related as shown in figure 3. So α_{B-L} must be smaller than 0.006 and the stability bound can be lowered maximally by about 1 GeV from the SM prediction.

6 Conclusions

The work is motivated by the recent discovery of 126 GeV Higgs-like particle and also non-discovery of the low energy supersymmetric particles. The LHC experiment as well as other precision experiments such as the B-factories have put stringent constraints on the physics beyond the SM. In particular, a large parameter region of the low energy supersymmetry has been already excluded.

In this paper, we take an alternative approach to the hierarchy problem, namely, instead of introducing a large set of particles like in the supersymmetric models, we follow the Bardeen's argument on the hierarchy problem and construct a model with the classical conformality. It connects the electroweak physics with the Planck scale physics. A minimal construction of such a model with phenomenological viability is the $B - L$ extension of the SM at the TeV scale. The model has a classical conformality, and scalar mass terms are absent. Therefore, the symmetries must be broken radiatively via the Coleman-Weinberg mechanism.

The TeV scale $B - L$ model is the Occam's razor scenario to solve the hierarchy problem, the vacuum stability condition as well as the phenomenological viabilities. There are two reasons why the $B - L$ extension is necessary. The first reason is the dynamics of the symmetry breaking. Since the CW mechanism does not work within the SM, we need another sector to achieve the radiative symmetry breaking. The $B - L$ gauge interaction is minimal for this

purpose. Another reason is the phenomenology. It is also a minimal extension to explain the neutrino oscillations as well as the leptogenesis. The breaking scale of the $B - L$ sector in this model is required to be not much higher than the TeV scale in order to avoid large logarithmic corrections to the Higgs mass. In previous papers [28], we showed that the TeV scale $B - L$ breaking is compatible with the leptogenesis scenario if the masses of the right-handed neutrinos are almost degenerate and the resonant leptogenesis [29] can work.

The main analysis of the paper is the RGEs in section 5. Motivated by the 126 GeV Higgs-like particle, we assume that the Higgs has a flat potential at the UV scale (e.g., the Planck scale). We showed that a small negative value of the scalar mixing term between the Higgs H and the SM singlet scalar Φ is radiatively generated. Once the $B - L$ symmetry is spontaneously broken by the CW mechanism at the TeV scale, the radiatively generated mixing triggers the EWSB. The ratio between the two breaking scales is dynamically determined in terms of the gauge couplings. This gives another reason why the TeV scale is required from the EWSB. In most TeV scale $B - L$ models, the scale M_{B-L} of the $B - L$ symmetry breaking is just assumed by hand. In our model, however, it is determined from the relation (24) as

$$M_{B-L} \sim \frac{\sqrt{\lambda_H}}{c\alpha_{B-L}\alpha_Y} \times 246 \text{ GeV} \sim \frac{1}{\alpha_{B-L}} \times 35 \text{ GeV}. \quad (26)$$

and the mass of the $B - L$ gauge boson is given by

$$m_{Z'} = \sqrt{16\pi\alpha_{B-L}} M_{B-L} \sim \frac{1}{\sqrt{\alpha_{B-L}}} \times 250 \text{ GeV}. \quad (27)$$

The dynamics of the model is essentially controlled by a single parameter, and it has a high predictability. If an extra $U(1)$ gauge boson and a SM singlet scalar are found in the future, the prediction of our model is the mass relation (20), e.g.,

$$m_\phi \sim 0.1 m_{Z'} \quad (28)$$

for $\alpha_{B-L} \sim 0.005$. The CW mechanism in the $B - L$ sector predicts a lighter SM singlet Higgs boson than the extra $U(1)$ gauge boson. It is different from the ordinary TeV scale $B - L$ model where the symmetry is broken by a negative squared mass term.

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<http://research.kek.jp/group/riron/workshop/theory2012/talkfiles/iso.pdf>

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Appendix A

We list the 1-loop renormalization group equations of various coupling constants in the $B-L$ model [30]. In the body of the paper, we also use $\alpha_{\lambda,a} = \lambda_a/4\pi$ for the scalar quartic couplings, $\alpha_A = g_A^2/4\pi$ for the gauge couplings and $\alpha_N^i = (Y_N^i)^2/4\pi$ for the Yukawa couplings.

The RGEs of the gauge couplings are given by

$$\frac{dg_Y}{dt} = \frac{1}{16\pi^2} \left[\frac{41}{6} g_Y^3 \right], \quad (29)$$

$$\frac{dg_{B-L}}{dt} = \frac{1}{16\pi^2} \left[12g_{B-L}^3 + 2\frac{16}{3}g_{B-L}^2g_{mix} + \frac{41}{6}g_{B-L}g_{mix}^2 \right], \quad (30)$$

$$\frac{dg_{mix}}{dt} = \frac{1}{16\pi^2} \left[\frac{41}{6}g_{mix} (g_{mix}^2 + 2g_Y^2) + 2\frac{16}{3}g_{B-L} (g_{mix}^2 + g_Y^2) + 12g_{B-L}^2g_{mix} \right]. \quad (31)$$

The RGEs for the Yukawa couplings are

$$\frac{dY_t}{dt} = \frac{1}{16\pi^2} Y_t \left(\frac{9}{2} Y_t^2 - 8g_3^2 - \frac{9}{4} g^2 - \frac{17}{12} g_Y^2 - \frac{17}{12} g_{mix}^2 - \frac{2}{3} g_{B-L}^2 - \frac{5}{3} g_{mix} g_{B-L} \right), \quad (32)$$

$$\frac{dY_N}{dt} = \frac{1}{16\pi^2} Y_N \left(Y_N^2 + \frac{1}{2} Tr [Y_N^2] - 6g_{B-L}^2 \right). \quad (33)$$

Finally the RGEs for the scalar quartic couplings are given by

$$\begin{aligned} \frac{d\lambda_H}{dt} = \frac{1}{16\pi^2} & \left(24\lambda_H^2 + \lambda_{mix}^2 - 6Y_t^4 + \frac{9}{8}g^4 + \frac{3}{8}g_Y^4 + \frac{3}{4}g^2g_Y^2 + \frac{3}{4}g^2g_{mix}^2 + \frac{3}{4}g_Y^2g_{mix}^2 + \frac{3}{8}g_{mix}^4 \right. \\ & \left. + \lambda_H (12Y_t^2 - 9g^2 - 3g_Y^2 - 3g_{mix}^2) \right), \quad (34) \end{aligned}$$

$$\frac{d\lambda_\Phi}{dt} = \frac{1}{16\pi^2} \left(20\lambda_\Phi^2 + 2\lambda_{mix}^2 - \frac{1}{2} Tr [Y_N^4] + 96g_{B-L}^4 + \lambda_\Phi (2Tr [Y_N^2] - 48g_{B-L}^2) \right), \quad (35)$$

$$\begin{aligned} \frac{d\lambda_{mix}}{dt} = \frac{1}{16\pi^2} & \left[\lambda_{mix} \left(12\lambda_H + 8\lambda_\Phi + 4\lambda_{mix} + 6Y_t^2 - \frac{9}{2}g^2 - \frac{3}{2}g_Y^2 - \frac{3}{2}g_{mix}^2 + Tr [Y_N^2] - 24g_{B-L}^2 \right) \right. \\ & \left. + 12g_{mix}^2g_{B-L}^2 \right]. \quad (36) \end{aligned}$$

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